Comparison of Three Methods for the Spatial Interpolation of Rainfall Data

Study Project

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Declaration of Authorship

I, Yanan Cao, declare that this study project, titled “Comparison of Three Methods for the Spatial Interpolation of Rainfall Data”, and the work presented in here are based on my own effort, except where otherwise acknowledged. This study project has not been presented previously to any other examination board or publications.

Signed:

Date:
Abstract

Accurate spatially distributed rainfall data are essential for many applications including hydrological modelling and water resources management etc. Spatial interpolation of measurements from point-base gauge stations is a common way to obtain spatially distributed rainfall data. In this study project, three interpolation methods, Inverse Distance Weighting (IDW), Ordinary Kriging, and Universal Kriging, were applied and evaluated in a case study area in Jiangxi Province, China. This report gives a detailed introduction of each interpolation method. Then three interpolation methods were applied to generate spatially distributed rainfall map based on 63 rain gauge stations in the study area. The study was carried out in R Studio, including processing of the data, validation and visualization. The Leave-One-Out-Cross-Validation was applied and three commonly used metrics, Root mean square error (RMSE), coefficient of determination ($r^2$), and Nash-Sutcliffe efficiency (NSE), which were calculated for assessing the performance of three interpolation methods. Monthly and seasonal analysis of the generated rainfall from three interpolation methods were analyzed and compared using the three metrics as well as visual analysis. It can be concluded that the Ordinary Kriging slightly outperforms the other two interpolation methods in terms of metrics and visual analysis. However, a further statistical analysis showed that the differences among the three interpolation methods are not statistically significant. Furthermore, a zonal statistical analysis was applied to show whether the differences between each two interpolation methods are significant using the unpaired two-sample-t-test. The t-test revealed that the differences are still not statistically significant with a high p value. Therefore, considering the fact that both Kriging methods require more complex computation and intensive computational time but both did not generate significantly improved interpolation results than the simple IDW method, this study project identifies
the IDW as a better practical method for spatial interpolation of monthly rainfall in the study area.
Acknowledge

This study project has been completed with considerable guidance and assistance from many individuals.

I would like to express my gratitude to Dr. Zheng Duan from the Chair of Hydrology and River Basin Management, Technical University of Munich, for introducing the fundamental theories and application in R Programming for spatial interpolation in rainfall and for his contribution and advices as a supervisor in the course of this study project. This report has also benefited from the valuable lessons provided in the lecture “Remote Sensing in Hydrology” by Dr Zheng Duan, held in the summer term of 2016 at the Technical University of Munich.
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1. Introduction

Accurate spatially distributed rainfall data are often required for many applications including hydrological modelling and water resources management (Wagner, et al., 2012). Generally, the readily available rainfall measurements are provided from point-base rain gauges. The rain gauges tend to be unevenly and sometimes sparsely scattered in the observed area, and the amount and location are limited by unfeasible installation and high expense due to various geographical factors. Spatial interpolation of measurements from point-base gauge stations is a common way to obtain spatially distributed rainfall data. A range of different methods of interpolation have been proposed. Generally they can be classified to three main categories: non-geostatistical interpolators, geostatistical interpolators and combined method (Li & Heap, 2008). The commonly used methods include IDW (Inverse Distance Weight), Spline, Ordinary Kriging, Universal Kriging, etc. Many studies have been done in different regions to find the most suitable interpolator to produce the most accurate spatially distributed rainfall data.

Table 1-1 presents a summary of relevant literature on evaluation of spatial interpolation, with data used, location and size of the study area as well as the key results. The studies considered as many as hundreds of stations covering up to 84000 km², using daily, monthly or annual rainfall data. These studies differ in many aspects, such as the number of used rain gauge stations, the interpolation methods evaluated, and temporal scales (hourly, daily, monthly or annual) at which evaluation was conducted. One key conclusion can be drawn from this literature review as reflected in Table 1-1, that is, the performance of a certain interpolator varies from regions and regions depending on many factors. Therefore, it is indeed difficult to determine which interpolation method is the best suitable one in a certain study area of interest considering the large difference in feasibility, applicability, and accuracy between different types of
interpolation methods under different circumstances. For instance, the Ordinary Kriging was found to be the best one among all six methods for interpolation of annual rainfall in East of Nebraska and the northern Kansas (Tabios III & Salas, 1985). Pierre Goovaerts attained the similar result based on daily rainfall in a sparsely distributed area that Ordinary Kriging yields the most accurate prediction among the six techniques applied in a 5000 km² region of Portugal with 36 stations (Goovaerts, 2000). Antonino Di Piazza revealed the similar conclusion that from the comparison of several univariate methods, Ordinary Kriging was proved to obtain the best performance as well (Di Piazza, et al., 2011).

Table 1-1 Summary of relevant literature on evaluation of spatial interpolation

<table>
<thead>
<tr>
<th>STUDY</th>
<th>DATA</th>
<th>LOCATION/ SIZE OF STUDY AREA</th>
<th>KEY RESULTS</th>
</tr>
</thead>
</table>
| (TABIOS III & SALAS, 1985)   | Annual rainfall data at 29 rain gauge stations in time period of 1931-1960 | East of Nebraska and some in the northern Kansas/ 52,000 km² | 1. The Kriging techniques are the best among all the techniques analyzed.  
2. Polynomial interpolation gives the poorest results.  
3. The IDW and Thiessen polygon methods give similar results, however, the former generally gives smaller error of interpolation. |
| (GOOVAERTS, 2000)            | Daily rainfall data recorded at 36 stations in the time period of January 1970 – March 1995 | Algarve region (Portugal)/ 5000 km² | 1. RMSE of Kriging prediction is up to half the error produced using inverse square distance.  
2. Cross validation has shown that prediction performances can vary greatly among algorithms.  
3. Ordinary Kriging which ignores elevation is in fact better than linear regression |
when the correlation is smaller than 0.75.
4. Co-Kriging maps show less details than the SKlm and KED maps that are greatly influenced by the pattern of the DEM.

| (HABERLANDT, 2007) | Daily rainfall data from 281 non-recording stations, hourly data from 21 recording stations | South-East-Germany/ 25000 km² | 1. Using all additional information simultaneously with KED gives the best performance.
2. The impact of the semivariogram on interpolation performance is not very high.
3. The exclusive use of uncalibrated radar data cannot be recommended, because this results in a significant underestimation of rainfall. |
2. Thiessen method obviously provides an unrealistic discontinuous rain field.
3. The areal-mean method clearly does not show the true spatial variation of rainfall.
4. The inverse-distance method is the most appropriate for this case. |
2. RIDW and RK perform similarly well, while RIDW is less complex.
3. Cross-validation is not sufficient to identify the most |
<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>(BARGAOUI &amp; CHEBBI, 2008)</td>
<td>Two extreme events which are highly cumulated rainfall amounts on a large scale, 1973 event: 13 instantaneous rain gauges, 1986 event: 8 stations.</td>
</tr>
<tr>
<td>(DI PIAZZA, ET AL., 2011)</td>
<td>Monthly and annual rainfall data from 247 rain gauges in the time period of January 1921 – December 2004.</td>
</tr>
<tr>
<td>(HAYLOCK, ET AL., 2008)</td>
<td>Daily rainfall data from about 250 stations, covering the time period 1950-2006.</td>
</tr>
</tbody>
</table>
2. The y-IDW interpolation is better in correlation and efficiency but worse in bias.  
3. Time series performance is better than spatial performance, due to the scale of data.  
4. The application of a statistical distance measure between neighbored rainfall time series instead of geographical distances between stations slightly improves averaged interpolation performance. |

Studies have been shown that IDW and Kriging methods perform better in sparsely distributed areas such as in the study of Tabios and Salas’ (Tabios III & Salas, 1985), Pierre Goovaerts’ (Goovaerts, 2000), and Antonino Di Piazza’s (Di Piazza, et al., 2011).

Additionally, validation and visualization of the main types of interpolation based on a real case will give a general perspective about rainfall modeling, as well as the performance testing. After analysis and evaluation of different interpolation methods, further exploration of estimation and assessment will allow a deeper thinking of optimization of existing approaches according to different situation. Therefore, the objective of this study project is to compare and evaluate three commonly used interpolation methods (IDW, Ordinary Kriging, and Universal Kriging) in a sub-catchment of the Ganjiang River catchment in Jiangxi Province, China, which has a network of 63 rainfall stations in an area of 17000 km². This study will provide a valuable guidance on the selection of suitable interpolation method for relevant applications in the local community.
2. Study area and rainfall gauge data

This study has been carried out for a sub-catchment of the Ganjiang River catchment, which flows through the western part of Jiangxi province, China, before flowing into Lake Poyang and thence into the Yangtze River. Ganjiang River is the longest river in Jiangxi Province, China, with a total length of 991 kilometers, and a surface area of 83,500 km$^2$. Climate in the Ganjiang River catchment is mild, with adequate rainfall. The average annual rainfall is 1400-1800 mm/year. The May-June months of rainfall are concentrated, with more floods. The March-August months takes 71% of total rainfall (Chen & Gao, 2003).

In this study, measured rainfall data from 63 rain gauge stations for the period 2001-2010 were obtained from the Hydrologic Yearbooks published by the Hydrographic Office of Jiangxi Province in China, with the area of about 17000 km$^2$. The locations of these rain gauge stations are shown in Fig. 2-1. During the 10 years, in the year of 2002, average annual rainfall of 63 stations reached the highest of 2347.75 mm/year. In the year of 2008, average annual rainfall of 63 stations was the medium of the whole dataset, 1544.02 mm/year. In the year of 2003, the average annual rainfall of 63 stations reached the lowest amount, i.e. 1050.45 mm/year, which is abnormal for the whole dataset. It was found that the the year of 2003 is had many missing rainfall data; specifically, the rainfall in July in 2003 was found to be only 20 mm/month on average, while rainfall in both June and August was over 150 mm/month. Therefore, the year of 2003 was excluded for the analysis in this study. Instead, the second lowest rainfall year, 2009, with an average annual rainfall of 1419.05 mm/year was considered. This study concentrated on these three typical years: 2002, 2008, and 2009, which serves to evaluate of different interpolation methods in wet, average and dry situations.
Fig. 2-1 Locations of study area and rain gauge stations
3. Methodology

Three different interpolation algorithms were compared in this study. They are IDW (Inverse Distance Weight), Ordinary Kriging, and Universal Kriging. Monthly rainfall data were used as the input data for interpolation. The background and principle of each interpolation method, processing in R programming, and evaluation method are described in the following subsections.

3.1 IDW

The Inverse Distance Weighting interpolator assumes that each input point has a local influence that diminishes with distance. It weights the points closer to the processing cell greater than those further away. A specified number of points, or all points within a specified radius can be used to determine the output value of each location. Use of this method assumes the variable being mapped decreases in influence with distance from its sampled location (Lang, 2015).

The rainfall value \( z \) can be estimated as a linear combination of several surrounding observations, with the weights being inversely proportional to the square between observations and \( x_0 \):

\[
\hat{z}(x_0) = \sum_{i=1}^{N} \lambda_i z(x_i)
\]

where the weights \( \lambda_i \) are expressed as function of distance as following:

\[
\lambda_i = \frac{d_{i0}^{-r}}{\sum_{i=1}^{N} d_{i0}^{-r}}
\]

The basis idea for IDW method is that observations that are close to each other on the ground tend to be more similar than those further apart, hence observations closer to \( x_0 \) receive a larger weight. This exact interpolation method requires the choice of the exponent \( r \) and of a search radius \( R \) or alternatively the minimum number \( N \) of points required for the interpolation (Di Piazza, et al., 2011). Here in the study, \( N \) is the number of measured sample points surrounding the prediction location that will be used in the prediction,
which is 63. As for the power parameter $r$, it influences the weighting of the measured location’s value on the prediction location’s value. As the distance increases between the measured sample locations and the prediction location, the influence that the measured point will have on the prediction will decrease exponentially. Using a power parameter of 2 for daily and monthly time steps, 3 for hourly and 1 for yearly would appear to minimize the interpolation errors (Dirks, et al., 1998). Furthermore, this power d is usually set to 2, following Goovaert (2000) and Lloyd (2005). Therefore, inverse square distances are used in the estimation. Consequently, a power value of 2 was chosen for IDW in this study.

3.2 Ordinary Kriging

Kriging, also named as Gaussian process regression, is using Gaussian process to give the most linear unbiased solution. Kriging is firstly raised by a South African geologist, D.G. Krige, in 1950s. Later in 1962, the term “Kriging” and the formalism of this method are coined and systematized by a French mathematician, Georges Matheron (Ly, et al., 2011). Since Kriging interpolation is realized based on a set of the point samples in entire area according to their spatial dependence structures, the results turn out to be more objective. Meanwhile, the range of error can be identified via error contour lines. However, disadvantage also remains due to the requirement of large amount of point samples. Ordinary Kriging, Simple Kriging, Co-Kriging, and Universal Kriging are main commonly used Kriging methods in recent days.

Ordinary Kriging, using semi-variogram instead of Euclidean distance, which is typical of IDW method, in order to measure the dissimilarity between observations and to assess the weights $\lambda_o(i)$, which are optimized based on the information that is inherent in the measured data. The weights can be obtained by solving the system below:

\[
\sum_{i=1}^{n} \lambda_o(i) \gamma(x_i, x_j) + \varnothing = \gamma(x_j, x_0) \quad \text{for all } j \\
\sum_{i=1}^{n} \lambda_o(i) = 1, 
\]

(4-3)
where $\gamma(x_i, x_j)$ stands for the value of the semi-v variogram function for the distance between the points $x_i$ and $x_j$, $\gamma(x_j, x_0)$ is the value for the distance between $x_j$ and the estimated location $x_0$, and $\emptyset$ is the Lagrange parameter. The semi-v variogram function can be derived by fitting a semi-v variogram model to the empirical semi-v variogram, which is calculated for all distances $h$ through following equation:

$$\gamma(h) = \frac{1}{2n} \sum_{i=1}^{n} [z(x_i) - z(x_i + h)]^2$$  \hspace{1cm} (4-4)

In the study, Ordinary Kriging should be applied based on fitting semi-v variogram with experimental parameters for every month of the 10-year period, then get the most accurate semi-v variogram model through iteration. An example of using the experimental semi-v variogram i.e. the equation 4-4 to fit a new semi-v variogram is shown in the below graph (Fig. 3-1), applied the monthly rainfall data of July, 2002.

Fig. 3-1 Sample semi-v variogram of monthly rainfall (July, 2002) with the fitted model: the experimental semi-v variogram, i.e. equation 4-4
### 3.3 Universal Kriging

Universal Kriging, also called Kriging with a trend (KT), was firstly proposed by Matheron in 1969 (Armstrong, 1984). It is an extension of Ordinary Kriging by adding a local trend within the neighbor area as a smoothly varying function of the coordinates (Li & Heap, 2008). Universal Kriging assumes a general linear trend model. It includes the drift functions to calculate \( z(x_0) \), which is the expectation of \( Z(x_0) \). Considering

\[
z(x_0) = a_0 + a_1 u + a_2 v + a_3 u^2 + a_4 u v + a_5 v^2
\]  

(4-5)

where \( u, v \) are the coordinates of point \( x_0 \). Then we can get

\[
\sum_i \lambda_i^x (a_0 + a_1 x_i + a_2 y_i + a_3 x_i^2 + a_4 u v + a_5 v^2) = a_0 + a_1 x_0 + a_2 y_0 + a_3 x_0^2 + a_4 x_0 y_0 + a_5 y_0^2
\]

(4-6)

In order to set up this equation, the following equations can be acquired

\[
\begin{align*}
\sum_i \lambda_i^x & = 1; \quad \sum_i \lambda_i^x x_i = x_0 \\
\sum_i \lambda_i^y y_i & = y_0; \quad \sum_i \lambda_i^x x_i^2 = x_0^2 \\
\sum_i \lambda_i^x x_i y_i & = x_0 y_0; \quad \sum_i \lambda_i^y y_i^2 = y_0^2
\end{align*}
\]

(4-7)

set

\[
\sum_i \lambda_i^x P_l(x_i) = P_l(x_0), \quad (l = 0, 1, 2, 3, 4, 5)
\]

(4-8)

in which \( P_l = \{1, x, y, x^2, xy, y^2\} \).

As

\[
E [(Z_0^* - Z_0)^2] = Var(Z_0) + \sum_i \sum_j \lambda_i^x \lambda_j^y c(x_i, x_j) - 2 \sum_i \lambda_i^x c(x_i, x_0)
\]

(4-9)

where

\[
c(x_i, x_j) = COV(Z_i, Z_j)
\]

(4-10)

and it is based on Lagrange multiplier rule, we have

\[
\begin{align*}
\sum_i \lambda_i^x c(x_i, x_0) - \sum_i \mu_i P_l(x_i) = c(x_i, x_0) & \quad (i = 1, 2, ..., n) \\
\sum_i \lambda_i^x P_l(x_i) = P_l(x_0) & \quad (l = 0, 1, ..., 5)
\end{align*}
\]

(4-11)

which could be rewritten in the matrix form such as \( Ax = b \) to calculate the value of \( \lambda_i^x (i = 1, 2, ..., n) \). From the first equation, finally we could get the estimation of the unknown points. And the sample number is \( N = 63 \) in the study. Similar semi-variogram fitting procedures were applied on every month’s
rainfall data as Ordinary Kriging.

3.4 Implementation and evaluation of three interpolation using R Programming

In this study, all three interpolation methods were implemented and evaluated using R programming. The visualization was performed to allow for a clearer and more feasible sight for comparative analysis. The performance of each interpolation method was evaluated via error estimation, which provides quantitative description of accuracy comparison. The following subsections present details on processing in R studio.

3.4.1 Raw data collection and pre-processing

The used rain gauge data were available as Excel spreadsheets, and prepared in the format of scattered points.

Raw point data was extracted as comma-separated values (CSV) using Visual Basic Application in Excel, with only readable contents of monthly and yearly rainfall data. The format was more applicable for R programming to deal with, and prepared for the different types of interpolation.

At the same time, the location data, which was originally in the format of shapefile, was imported in R programming then transformed to spatial points data frame with the information of coordinates in longitude and latitude. When applying R, Universal Transverse Mercator (UTM) coordinate system works better than latitude-longitude coordinates system. Therefore, the location data was transformed again into a UTM system, within a grid of 1km*1km.

Then the rain fall data was combined with location data to set up a whole new spatial point data frame with the both information.

3.4.2 Implementation of three Interpolation methods

The three interpolation methods were applied in R programming through different tools and packages.

IDW interpolation was realized through the tool “idw” in the package “gstat”. By simply giving the rainfall values of each month, the table of coordinates with
the 63 rain gauge stations, the created grid covering every points (established in step 3.4.1), and the power value of 2, the interpolation can be processed within several seconds. The output contains predicted values of all points in the grid, i.e. 17286 points.

Ordinary Kriging interpolation used firstly the tools “variogram” from the package “gstat” to calculate the isotropic empirical semi-variogram of two-dimensional rainfall data for visualizing stationary (time-averaged) autocorrelation structure. Then fit the sill and range from the experimental model to the original semi-variogram using “fit.variogram” tool from the same package. After that, the suitable model with a more accurate sill and range can be drawn out. In next step, the suitable model was iterated to the original semi-variogram. Using the tool “gstat”, interpolation can be applied based on the model. Lastly, through the tool “predict”, the predictions were obtained from the fitted model object.

For Universal Kriging, the only difference from the process of Ordinary Kriging is that when defining the dependent variables as a linear model of independent variables, suppose the dependent variable has the name z, the formula should be set to z~1 for Ordinary Kriging, and for Universal Kriging, suppose z is linearly dependent on the coordinates, x and y, use the formula z~x+y, as defined in the package “gstat” (Pebesma & Graeler, 2017).

3.4.3 Evaluation of interpolated rainfall

Evaluation of interpolated rainfall can be performed by comparing the interpolated values with measurements from rain gauge station. However, it is not advisable to compare the predictive accuracy of a set of models using the same observations used for estimating the models. Therefore, an independent set of data (the test sample) should be used. Then, the model showing the lowest error on the test sample (i.e., the lowest test error) is identified as the best one in this study area.

The most commonly used evaluation method is the cross validation to
validate the accuracy of an interpolation method, achieved by eliminating information (Voltz M., 1990). Since cross validation solves the inconvenience of redundant data collection, all collected data can be used for later estimation (Webster R., 2001). In its basic version, k-fold cross-validation, the samples are randomly partitioned into k sets of roughly equal size. A model is fit using all the samples except the first subset. Then, the prediction error of the fitted model is calculated using the first held-out samples. The same operation is repeated for each fold and the model’s performance is calculated by averaging the errors across the different test sets. K is usually fixed at 5 or 10. Cross-validation provides an estimate of the test error for each model. Cross-validation is one of the most widely-used method for model selection, and for choosing tuning parameter values.

The case where k=n corresponds to the so called leave-one-out cross-validation (LOOCV) method. In this case the test set contains a single observation. The advantages of LOOCV are: 1) it doesn’t require random numbers to select the observations to test, meaning that it doesn’t produce different results when applied repeatedly, and 2) it has far less bias than k-fold CV because it employs larger training sets containing n−1 observations each. On the other side, LOOCV presents also some drawbacks: 1) it is potentially quite intense computationally, and 2) due to the fact that any two training sets share n−2 points, the models fit to those training sets tend to be strongly correlated with each other.

Therefore, in this study LOOCV was applied to evaluate the performance of three interpolation algorithms. The related tools in R programming that were used to perform the evaluation include “vector” in the package of “base”, which help calculate out predicted value on every point using observed values on all other 62 points based on IDW interpolation, and the specific cross validation tool for Kriging interpolators, “krige.cv” from the package of “gstat”.

3.4.4 Visualization of data

Interpolated maps will be created via plotting tool in R programming based on the same calibration points, which will enable a more significant view of analysis under the same discipline. The visual analysis will screen the data values to identify the incorrect and illogical spatial information (Robinson & Metternicht, 2006).

3.4.5 Error estimation

Three commonly used metrics: root mean square error (RMSE), coefficient of determination ($r^2$), and Nash-Sutcliffe efficiency (NSE) were calculated in order to allow a quantitative description of error estimation when applying different interpolation methods to the same region. The equations are described as:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(o_i-p_i)^2}{n}} \tag{4-12}$$

$$r^2 = 1 - \frac{\sum_{i=1}^{n}(o_i-p_i)^2}{\sum_{i=1}^{n}(o_i-o_m)^2} \tag{4-13}$$

$$\text{NSE} = 1 - \frac{\sum_{i=1}^{n}(p_i-o_i)^2}{\sum_{i=1}^{n}(o_i-o_m)^2} \tag{4-14}$$

where $n$ is the number of observations, $o$ is the observed value, $p$ is the predicted value, and $o_m$ is the mean of the observed value. Note that the sum is over all points of the interpolation grid.
4. **Results and discussion**

In this chapter, the results obtained using three different interpolation methods are analyzed and discussed.

4.1 *Monthly analysis*

Fig. 4-1 shows the average monthly rainfall from all 63 rain gauge stations in the three typical years, 2002, 2008 and 2009. It can be seen that June, 2002 is with the highest rainfall, while October, 2009 the lowest rainfall.

![Monthly rainfall amount (mm/month) in 2008, 2009 and 2002](image)

In the year of 2009, which is with the lowest rainfall, IDW, Ordinary Kriging, and Universal Kriging can be applied to sketch out diagrams in following graphs, Fig. 4-2 and Fig. 4-3. After applying to cross validation, error estimations are shown below in Table 4-1.

The range of the RMSE values for three interpolation methods is 1.14 – 46.72 mm/month for IDW, 0.89 – 45.71 mm/month for Ordinary Kriging, and
0.98 – 45.71 mm/month for Universal Kriging. The average RMSE for three methods are 22.69, 22.10, and 22.78 mm/month, respectively. Therefore, the Ordinary Kriging gave the lowest error, while Universal Kriging gave the highest error in terms of RMSE.

R² values for three interpolation methods’ ranges are 0.02 – 0.56 for IDW, 0.03 – 1.00 for Ordinary Kriging, and 0.03 – 1.00 for Universal Kriging. The average r² for three methods are 0.34, 0.43, and 0.38. Therefore, Ordinary Kriging method is most predictable, while Universal Kriging holds the lowest.

When analyzing the NSE values, the ranges are -0.01 – 0.51 for IDW, -0.03 – 0.58 for Ordinary Kriging, and -0.03 – 0.58 for Universal Kriging. While average values are 0.30, 0.34 and 0.28. Therefore, Ordinary Kriging shows the highest credibility, while Universal Kriging shows the lowest.

RMSE values revealed here are not sufficient to prove the Ordinary Kriging to be the best selection, since the three methods all hold relatively high errors. Sarann Ly compared IDW, Ordinary Kriging, Universal Kriging and other interpolators based on different numbers of rain gauges and pointed out that estimates based on more rain gauges tended to produce lower RMSE values, ranging from 8.5 mm for 4 gauges to 2.5 mm for 70 gauges (Ly, et al., 2011). Moreover, in Paul Wagner’ research, comparisons between seven interpolators including IDW, Ordinary Kriging, and Universal Kriging based on RMSE estimation showed that the mean RMSE values were 9.8-12.3 mm, much lower than the results revealed in this study (Wagner, et al., 2012).

There were little differences between the three methods according to these three evaluations. In the two studies mentioned above, though the differences among the three interpolators were not obvious as well, especially for RMSE values, RMSE and NSE still were used as indicators for the best performance of studied interpolation methods. Therefore, in this study, based on monthly analysis in the year of 2009, Ordinary Kriging outperforms the other two interpolation methods with a relatively higher accuracy and credibility, regarding to its RMSE and NSE values.
Fig. 4-2 Interpolated monthly rainfall from three different methods (IDW, OK and UK) for the period January to June, 2009
Fig. 4-3  Interpolated monthly rainfall from three different methods (IDW, OK and UK) for the period July to December, 2009
Table 4-1 Validation results for three interpolation methods in each month of the year of 2009 (RMSE in mm/month)

<table>
<thead>
<tr>
<th>Month</th>
<th>IDW</th>
<th>OK</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>JANUARY</td>
<td>RMSE</td>
<td>5.26</td>
<td>5.14</td>
</tr>
<tr>
<td></td>
<td>$r^2$</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>NSE</td>
<td>-0.01</td>
<td>0.03</td>
</tr>
<tr>
<td>FEBRUARY</td>
<td>RMSE</td>
<td>5.82</td>
<td>5.9</td>
</tr>
<tr>
<td></td>
<td>$r^2$</td>
<td>0.17</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>NSE</td>
<td>0.16</td>
<td>0.13</td>
</tr>
<tr>
<td>MARCH</td>
<td>RMSE</td>
<td>15.03</td>
<td>14.31</td>
</tr>
<tr>
<td></td>
<td>$r^2$</td>
<td>0.45</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>NSE</td>
<td>0.41</td>
<td>0.46</td>
</tr>
<tr>
<td>APRIL</td>
<td>RMSE</td>
<td>25.35</td>
<td>25.27</td>
</tr>
<tr>
<td></td>
<td>$r^2$</td>
<td>0.24</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>NSE</td>
<td>0.22</td>
<td>0.23</td>
</tr>
<tr>
<td>MAY</td>
<td>RMSE</td>
<td>28.95</td>
<td>26.2</td>
</tr>
<tr>
<td></td>
<td>$r^2$</td>
<td>0.51</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>NSE</td>
<td>0.46</td>
<td>0.56</td>
</tr>
<tr>
<td>JUNE</td>
<td>RMSE</td>
<td>44.29</td>
<td>44.73</td>
</tr>
<tr>
<td></td>
<td>$r^2$</td>
<td>0.39</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>NSE</td>
<td>0.37</td>
<td>0.35</td>
</tr>
<tr>
<td>JULY</td>
<td>RMSE</td>
<td>42.01</td>
<td>45.71</td>
</tr>
<tr>
<td></td>
<td>$r^2$</td>
<td>0.13</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>NSE</td>
<td>0.13</td>
<td>-0.03</td>
</tr>
<tr>
<td>AUGUST</td>
<td>RMSE</td>
<td>46.72</td>
<td>40.68</td>
</tr>
<tr>
<td></td>
<td>$r^2$</td>
<td>0.51</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>NSE</td>
<td>0.44</td>
<td>0.58</td>
</tr>
<tr>
<td>SEPTEMBER</td>
<td>RMSE</td>
<td>26.73</td>
<td>27.68</td>
</tr>
<tr>
<td></td>
<td>$r^2$</td>
<td>0.23</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>NSE</td>
<td>0.21</td>
<td>0.16</td>
</tr>
<tr>
<td>OCTOBER</td>
<td>RMSE</td>
<td>1.14</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>$r^2$</td>
<td>0.34</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>NSE</td>
<td>0.26</td>
<td>0.46</td>
</tr>
<tr>
<td>NOVEMBER</td>
<td>RMSE</td>
<td>17.22</td>
<td>15.62</td>
</tr>
<tr>
<td></td>
<td>$r^2$</td>
<td>0.48</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>NSE</td>
<td>0.44</td>
<td>0.54</td>
</tr>
<tr>
<td>DECEMBER</td>
<td>RMSE</td>
<td>13.75</td>
<td>12.92</td>
</tr>
<tr>
<td></td>
<td>$r^2$</td>
<td>0.56</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>NSE</td>
<td>0.51</td>
<td>0.57</td>
</tr>
</tbody>
</table>
In January, February and October, which are the lowest rainfall months through the year, the average of rainfall of which are 35.03mm, 14.08mm and 1.96mm, respectively. In the three months, Ordinary Kriging shows a lower deviation and a better performance with a higher credibility based on RMSE and NSE values. The following figure (Fig. 4-4) gives a graphic comparison of the three different interpolators. With the same resolution, Ordinary Kriging and Universal Kriging presents more continuously than IDW.

![Interpolated monthly rainfall from three different methods (IDW, OK and UK) for October, 2009](image)

In the year of 2002, which is with the biggest rainfall, IDW, Ordinary Kriging, and Universal Kriging can be applied to sketch out diagrams in below graphs (Fig. 4-5, Fig. 4-6). After applying to cross validation, error estimations are shown below in Table 4-2.

The RMSE values of three interpolation methods’ range are 8.59 – 84.89 mm/month for IDW, 8.15 – 61.61 mm/month for Ordinary Kriging, and 9.08 – 61.69 mm/month for Universal Kriging. And the average RMSE are 33.10, 30.85, and 30.97 mm/month. Ordinary Kriging is proved to have the lowest error again, while IDW has the highest.

R² values for three interpolation methods’ ranges are 0.03 – 0.82 for IDW,
0.01 – 0.83 for Ordinary Kriging, and 0.01 – 0.82 for Universal Kriging. The average $r^2$ for three methods are 0.35, 0.36, and 0.35. Therefore, Ordinary Kriging method is most predictable, while IDW and Universal Kriging holds the same lower $r^2$.

Fig. 4-5 Interpolated monthly rainfall from three different methods (IDW, OK and UK) for the period January to June, 2002
Fig. 4-6 Interpolated monthly rainfall from three different methods (IDW, OK and UK) for the period July to December, 2002
In 2002, the NSE ranges are -0.02 – 0.67 for IDW, -0.09 – 0.83 for Ordinary Kriging, and -0.08 – 0.82 for Universal Kriging. And the average NSE are 0.31, 0.34, and 0.33. Here Ordinary Kriging shows the highest credibility, and IDW shows the lowest.

With the similar results as in 2009, Ordinary Kriging presents highest accuracy and credibility, though the differences among three interpolation methods based on RMSE, $r^2$, and NSE are not very significant.

Table 4-2 Validation results for three interpolation methods in each month of the year of 2002 (RMSE in mm/month)

<table>
<thead>
<tr>
<th></th>
<th>IDW</th>
<th>OK</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>JANUARY</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>16.55</td>
<td>16.2</td>
<td>16.21</td>
</tr>
<tr>
<td>$r^2$</td>
<td>0.03</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>NSE</td>
<td>-0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td><strong>FEBRUARY</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>9.1</td>
<td>9.11</td>
<td>9.08</td>
</tr>
<tr>
<td>$r^2$</td>
<td>0.23</td>
<td>0.24</td>
<td>0.24</td>
</tr>
<tr>
<td>NSE</td>
<td>0.23</td>
<td>0.23</td>
<td>0.24</td>
</tr>
<tr>
<td><strong>MARCH</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>18.79</td>
<td>16.26</td>
<td>16.8</td>
</tr>
<tr>
<td>$r^2$</td>
<td>0.49</td>
<td>0.57</td>
<td>0.56</td>
</tr>
<tr>
<td>NSE</td>
<td>0.42</td>
<td>0.57</td>
<td>0.54</td>
</tr>
<tr>
<td><strong>APRIL</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>22.94</td>
<td>23.3</td>
<td>23.49</td>
</tr>
<tr>
<td>$r^2$</td>
<td>0.67</td>
<td>0.63</td>
<td>0.63</td>
</tr>
<tr>
<td>NSE</td>
<td>0.63</td>
<td>0.62</td>
<td>0.61</td>
</tr>
<tr>
<td><strong>MAY</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>31.8</td>
<td>29.54</td>
<td>29.41</td>
</tr>
<tr>
<td>$r^2$</td>
<td>0.49</td>
<td>0.54</td>
<td>0.54</td>
</tr>
<tr>
<td>NSE</td>
<td>0.46</td>
<td>0.53</td>
<td>0.54</td>
</tr>
<tr>
<td><strong>JUNE</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>84.89</td>
<td>61.61</td>
<td>61.69</td>
</tr>
<tr>
<td>$r^2$</td>
<td>0.82</td>
<td>0.83</td>
<td>0.82</td>
</tr>
<tr>
<td>NSE</td>
<td>0.67</td>
<td>0.83</td>
<td>0.82</td>
</tr>
<tr>
<td><strong>JULY</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>53.31</td>
<td>51</td>
<td>51.25</td>
</tr>
<tr>
<td>$r^2$</td>
<td>0.25</td>
<td>0.31</td>
<td>0.3</td>
</tr>
<tr>
<td>NSE</td>
<td>0.24</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td><strong>AUGUST</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>38.89</td>
<td>38.05</td>
<td>38.06</td>
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<tr>
<td>$r^2$</td>
<td>0.1</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>NSE</td>
<td>0.1</td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td><strong>SEPTEMBER</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>34.57</td>
<td>35.02</td>
<td>34.6</td>
</tr>
<tr>
<td>$r^2$</td>
<td>0.22</td>
<td>0.21</td>
<td>0.21</td>
</tr>
</tbody>
</table>
In June, which is with the highest rainfall through the year, and the three researched years as well, the average rainfall reaches 589.72mm. In the month, Ordinary Kriging still holds a higher accuracy and a better credibility based on RMSE and NSE values. The following figure (Fig. 4-7) shows that IDW interpolation obviously performs a lower continuity than Ordinary Kriging and Universal Kriging in the studied month.

![Fig. 4-7 Interpolated monthly rainfall from three different methods (IDW, OK and UK) for June, 2002](image)

When comparing the three methods in three years, more significant differences between Ordinary Kriging and Universal Kriging can be detected in March in 2009, December in 2009, and December in 2008, shown in the following figures (Fig. 4-8, Fig. 4-9, and Fig. 4-10). In each figure, Ordinary Kriging all shows a better continuity compared to Universal Kriging. Comparing
RMSE, $r^2$, and NSE between two methods in these three months, Ordinary Kriging always presents a higher accuracy, and a higher credibility as well.

Fig. 4-8 Interpolated monthly rainfall from two different methods (OK and UK) for March, 2009

Fig. 4-9 Interpolated monthly rainfall from two different methods (OK and UK) for December, 2009

Fig. 4-10 Interpolated monthly rainfall from two different methods (OK and UK) for December, 2008
4.2 Seasonal analysis

After evaluation of three interpolation methods in each month, it is also interesting to evaluate the seasonal performance of these methods. Therefore, rainfall data were divided into four seasons. Indicators used are RMSE and NSE in Fig. 4-11, combined with the average seasonal rainfall data in Table 4-3, conclusions can be drawn that

1) when in summer, which has the highest rainfall, deviation is the biggest among the whole year; while in winter, with the lowest rainfall, deviation reaches the the lowest;

2) across the whole year, Ordinary Kriging always shows relatively lower deviation and higher credibility than other two methods;

3) using different measure (RMSE, $r^2$, or NSE) may give different ranking of methods applied.

![Seasonal analysis: three interpolation performance as indicated by RMSE values (in mm/month) and NSE values](image)
Table 4-3 Average seasonal rainfall data based on three years

<table>
<thead>
<tr>
<th>Spring</th>
<th>Summer</th>
<th>Fall</th>
<th>Winter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mar, Apr, May</td>
<td>Jun, Jul, Aug</td>
<td>Sep, Oct, Nov</td>
<td>Dec, Jan, Feb</td>
</tr>
<tr>
<td>185.60 mm</td>
<td>242.68 mm</td>
<td>103.08 mm</td>
<td>58.73 mm</td>
</tr>
</tbody>
</table>

4.3 Zonal analysis

Temporal analysis presented above showed that Ordinary Kriging outperforms IDW and Universal Kriging, but to a minor extend. A zonal analysis to the three interpolation methods can statistically assess how different they present in the whole area. The zonal statistical analysis was to compute zonal average monthly rainfall value based on every interpolator for the zone that covers the complete grid. It was realized in R using the tool “zonal” from the package “raster”.

When applying zonal statistical analysis to the data for different methods in the three years, zonal average values based on the researched basin can demonstrate the differences of the interpolation results more obviously. Then unpaired two-sample-t-test was introduced to determine whether the means of two have significant difference from each other. The unpaired two-sample-t-test, also the ordinary non-sequential test in this study take the output rainfall data after interpolations from every two methods with means and unknown variance. It is desired to test the hypothesis that the two means are the same. A likelihood parameter p value is the probability that the hypothesis is true (Hajnal, 1961).

Table 4-4 shows the zonal average values and Table 4-5 presents the significance for the difference between each two methods in the three years. P values from t-tests are all above 98%, which suggests that the zonal interpolated rainfall data from three methods are not holding significant differences between each other.
### Table 4-4 T-tests results for zonal average values

<table>
<thead>
<tr>
<th></th>
<th>IDW</th>
<th>OK</th>
<th>UK</th>
<th>IDW</th>
<th>OK</th>
<th>UK</th>
<th>IDW</th>
<th>OK</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>JAN</td>
<td>101.65</td>
<td>100.04</td>
<td>100.12</td>
<td>63.64</td>
<td>64.58</td>
<td>64.68</td>
<td>35.00</td>
<td>35.08</td>
<td>35.08</td>
</tr>
<tr>
<td>FEB</td>
<td>46.49</td>
<td>46.69</td>
<td>46.70</td>
<td>78.49</td>
<td>79.46</td>
<td>79.53</td>
<td>13.72</td>
<td>14.03</td>
<td>14.02</td>
</tr>
<tr>
<td>MAR</td>
<td>156.10</td>
<td>154.06</td>
<td>153.94</td>
<td>170.69</td>
<td>171.22</td>
<td>167.44</td>
<td>181.89</td>
<td>180.06</td>
<td>180.38</td>
</tr>
<tr>
<td>APR</td>
<td>158.01</td>
<td>157.31</td>
<td>155.51</td>
<td>182.97</td>
<td>178.86</td>
<td>178.48</td>
<td>146.06</td>
<td>144.70</td>
<td>144.59</td>
</tr>
<tr>
<td>MAY</td>
<td>227.61</td>
<td>224.68</td>
<td>224.92</td>
<td>201.20</td>
<td>202.23</td>
<td>202.03</td>
<td>220.82</td>
<td>220.38</td>
<td>221.47</td>
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<tr>
<td>JUN</td>
<td>571.17</td>
<td>567.86</td>
<td>568.23</td>
<td>335.32</td>
<td>333.18</td>
<td>333.18</td>
<td>223.68</td>
<td>226.04</td>
<td>226.23</td>
</tr>
<tr>
<td>JUL</td>
<td>256.73</td>
<td>255.84</td>
<td>255.94</td>
<td>233.89</td>
<td>232.00</td>
<td>232.67</td>
<td>190.42</td>
<td>191.37</td>
<td>191.37</td>
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<tr>
<td>AUG</td>
<td>168.80</td>
<td>169.69</td>
<td>169.67</td>
<td>62.25</td>
<td>62.59</td>
<td>62.47</td>
<td>116.68</td>
<td>118.73</td>
<td>118.61</td>
</tr>
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<td>136.41</td>
<td>140.26</td>
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<td>82.11</td>
<td>80.88</td>
<td>80.92</td>
<td>58.80</td>
<td>57.85</td>
<td>57.90</td>
</tr>
<tr>
<td>OCT</td>
<td>310.77</td>
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<td>311.57</td>
<td>70.82</td>
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<td>71.45</td>
<td>1.94</td>
<td>1.99</td>
<td>1.99</td>
</tr>
<tr>
<td>NOV</td>
<td>40.18</td>
<td>40.92</td>
<td>41.07</td>
<td>55.96</td>
<td>57.42</td>
<td>56.97</td>
<td>154.55</td>
<td>153.33</td>
<td>153.44</td>
</tr>
<tr>
<td>DEC</td>
<td>103.86</td>
<td>103.10</td>
<td>103.14</td>
<td>6.95</td>
<td>6.91</td>
<td>6.84</td>
<td>70.34</td>
<td>71.27</td>
<td>71.08</td>
</tr>
</tbody>
</table>

### Table 4-5 Zonal average values for three methods in the three years

<table>
<thead>
<tr>
<th></th>
<th>IDW vs OK</th>
<th>IDW vs UK</th>
<th>OK vs UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002</td>
<td>99.35%</td>
<td>99.27%</td>
<td>99.92%</td>
</tr>
<tr>
<td>2008</td>
<td>99.36%</td>
<td>98.71%</td>
<td>99.34%</td>
</tr>
<tr>
<td>2009</td>
<td>99.81%</td>
<td>99.55%</td>
<td>99.73%</td>
</tr>
</tbody>
</table>

### 4.4 Discussion

Despite the not significant differences among the three interpolation methods, questions were raised for whether there exist significant differences among the evaluation metrics of the three methods. Therefore, the unpaired two-sample t-test was applied again to demonstrate the statistical significance of the differences between every two methods based on the three indicators applied in former chapters. The table below shows the p values of the t-test.
(Table 4-6) about three indicators based on three methods in three years. The t-test computes two series of two unpaired samples, each of which contains the evaluation results based on the monthly interpolated rainfall data using the indicators, RMSE, $r^2$, and NSE. Therefore, the result of t-test has one p value for every year using each indicator.

Table 4-6 T-test results for evaluation values in three years (RMSE in mm/month)

<table>
<thead>
<tr>
<th></th>
<th>2002</th>
<th>2008</th>
<th>2009</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMSE</td>
<td>IDW vs OK</td>
<td>IDW vs UK</td>
</tr>
<tr>
<td>2002</td>
<td>78.82%</td>
<td>79.84%</td>
<td>98.71%</td>
</tr>
<tr>
<td></td>
<td>90.41%</td>
<td>98.17%</td>
<td>91.08%</td>
</tr>
<tr>
<td></td>
<td>79.36%</td>
<td>86.07%</td>
<td>93.06%</td>
</tr>
<tr>
<td>2008</td>
<td>92.65%</td>
<td>98.86%</td>
<td>91.39%</td>
</tr>
<tr>
<td></td>
<td>32.08%</td>
<td>60.35%</td>
<td>68.44%</td>
</tr>
<tr>
<td></td>
<td>64.99%</td>
<td>78.90%</td>
<td>52.42%</td>
</tr>
<tr>
<td>2009</td>
<td>63.43%</td>
<td>70.54%</td>
<td>92.44%</td>
</tr>
<tr>
<td></td>
<td>84.76%</td>
<td>97.31%</td>
<td>82.97%</td>
</tr>
<tr>
<td></td>
<td>59.10%</td>
<td>70.50%</td>
<td>88.11%</td>
</tr>
</tbody>
</table>

P values are all above 5%, and especially the data of 2002 are nearly 100%, which means that the differences between every two methods applied are not significant. So the three interpolation methods have the similar results.

Therefore, when considering the workloads of the three methods, IDW should be selected instead of Ordinary Kriging, since two Kriging methods both took longer time and more work on iteratively fitting the semi-variograms.
5. Summary and conclusions

This study project made a comparative analysis of three different interpolation methods: IDW, Ordinary Kriging, and Universal Kriging, based on monthly rainfall data from 63 rain gauge stations in a case study area in Jiangxi Province, China. They perform differently facing different amount of rainfall data. Three years' rainfall data are considered in the study, including the richest rainfall year of 2002, the medium rainfall year of 2008, and the lowest rainfall year of 2009. When applying leave one out cross validation to the results of interpolations, difference can be drawn out according to RMSE, $r^2$, and NSE criteria. According to evaluation indicators based on monthly analysis, RMSE, $r^2$, and NSE, Ordinary Kriging consistently but slightly performs better than the other two methods. When referring to visualized output, Ordinary Kriging always outperforms IDW and Universal Kriging in both high and low rainfall months as well. Furthermore, according to seasonal analysis, Ordinary Kriging is holding the lowest deviation and the highest credibility through the four seasons.

Although Ordinary Kriging outperforms the other two methods, but the difference among the three methods based on the three indicators are not statistically significant. When applying unpaired two-sample-t-test to all the evaluation results for the three methods, not significant differences were detected with high p values. Moreover, the zonal analysis also suggests not significant differences lie among the three interpolated rainfall results.

Therefore, considering the computational time, it appears that the IDW is the practical better interpolation method in the investigated case study. It should be noted that due to time constraints, this study only investigated the performance of three interpolation methods for monthly rainfall. In future study, it would be interesting to evaluate more geostatistical interpolation methods such as the copulas (Bárdossy and Li, 2008), and particularly to evaluate the
performance of these methods for interpolation of rainfall at finer time scales (e.g. daily and hourly).
6. References


7. Appendix

Fig. 7-1 Interpolated monthly rainfall from three different methods (IDW, OK and UK) for the period January to June, 2008
Fig. 7-2 Interpolated monthly rainfall from three different methods (IDW, OK and UK) for the period July to December, 2008